SIMULATION OF PULSE WIDTH MODULATED (PWM) CHOPPER DRIVES USING Z-TRANSFORMATION

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ABSTRACT

In this paper, a class-C chopper circuit is modeled using z-transformation including the time variation of pulsewidth modulated (PWM) signal, and then a closed-loop speed control system using a direct current (dc) motor fed by the chopper is simulated. In the simulation, the entire system including dc motor, chopper, speed and current controllers are all modeled in z-domain. Simulation has been performed with the help of Matlab and Simulink. In addition, the closed-loop system has also been analyzed according to the average value modeling and the results obtained from the real-time simulation in z-domain are compared with the results obtained from the average value model.

1. INTRODUCTION

Although the dc machine is more expensive, the control principles and the converter equipment required are somewhat simpler compared to ac machines. The simplicity and flexibility of control of dc motors have made them suitable for adjustable speed drive applications. Also fast torque response has favored their use in high performance servo drives. Class-C type chopper is widely used especially in the speed control of dc motors in industry as a drive circuit. The dc motor fed by a chopper is usually modeled by the average value of the armature terminal voltage[1]. However this modeling does not include the ripples due to chopping of the armature voltage. In this paper the ripples due to chopping of the armature voltage are included into the model via the z-transformation. For this purpose, first PWM waveform at the output of the chopper has been estimated using the z-transformation and then the real-time simulation of the closed-loop speed control system considered has been carried out in z-domain entirely.

2. MODELING OF THE SYSTEM

Closed-loop speed control system in this study consists of a separately excited dc motor, chopper and speed and current controllers. Block diagram representation of the system is as given in Figure 1.

2.1. Modeling of DC Motor

The mechanical and electrical behaviour of a DC motor is described in continuous time domain by the following well known equations [2].

$$V_a = R_a.i_a + L_a.\frac{di_a}{dt} + E_g \quad (1)$$

$$E_g = K_a.\varphi.w \quad (2)$$

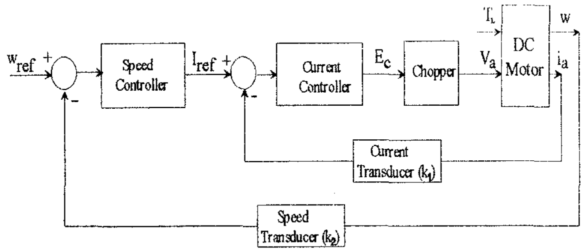

Figure 1. Block diagram of the dc motor speed control system

$$T_e = T_L + J\frac{dw}{dt} + B_v . w \tag{3}$$

$$T_e = K_a . \varphi . i_a \tag{4}$$

where,

V_a : Armature voltage (Volt)
R_a : Armature resistance (Ohm)
i_a : Armature current (Amper)
L_a : Armature inductance (Henry)
w : Motor speed (rad/sec)
E_g : Back electromotive force voltage (Volt)
$K_a\varphi$: Back electromotive force and torque constant (Volt/rd/sec or Nt-m/Amper)
T_e : Electromagnetic torque developed by the motor (Nt-m)
T_L : Load torque (Nt-m)
J : Total moment of inertia (kg-m^2)
B_v : Viscous friction constant (Nt-m/rd/sec).

Since, in this study, the field current is assumed constant, $K_a\varphi$ in Equation (2) and Equation (4) will be taken constant. The effect of armature current on the saturation has been neglected. Consequently, Equations (1) to (4) can be written in the state-space form as follows

$$\frac{d}{dt}\begin{bmatrix} i_a(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -K_a\varphi/L_a \\ K\varphi/J & -B_v/J \end{bmatrix} \cdot \begin{bmatrix} i_a(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} 1/L_a & 0 \\ 0 & -1/J \end{bmatrix} \cdot \begin{bmatrix} V_a(t) \\ T_L(t) \end{bmatrix} \tag{5}$$

or in the compact form

$$\frac{d}{dt}x(t) = A \cdot x(t) + B \cdot u(t) \tag{6}$$

and the output equation can be written as given below

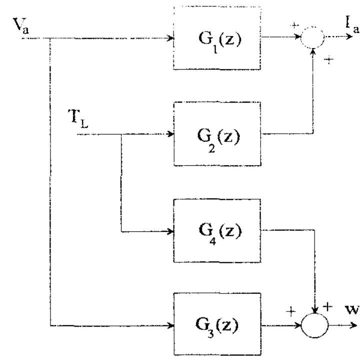

Figure 2. Separately excited dc motor model in z-domain

$$\begin{bmatrix} w(t) \\ i_a(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a(t) \\ w(t) \end{bmatrix} \tag{7}$$

or in the compact form

$$y = C.x \tag{8}$$

Therefore the transfer function matrix is

$$H(s) = C \cdot (sI - A)^{-1} \cdot B \tag{9}$$

Thus DC motor transfer functions in z-domain are obtained from the s-domain transfer function matrix H(s) using a function of Matlab Control System Toolbox which converts the continuous time system to discrete time. In other words DC motor can be represented as shown in Figure 2.

2.2 Modeling of the Chopper

In this study, a class-C chopper whose load voltage is positive whereas load current is both positive and negative is considered. The chopper circuit and its output PWM voltage waveform is given in Figure 3. In the circuit, the time t_{on} in which load is connected to input voltage can be changed via control signals i_{c1} and i_{c2}. In the closed-loop system, i_{c1} and i_{c2} are related to the current controller output signal (E_c) and the peak value of sawtooth signal (E_{sw}) as being seen in Figure 3-b, and the following equations can be written

$$t_{on} = \frac{T}{E_{sw}} E_c \qquad \text{for} \quad 0 \le E_c \le E_{sw} \tag{10}$$

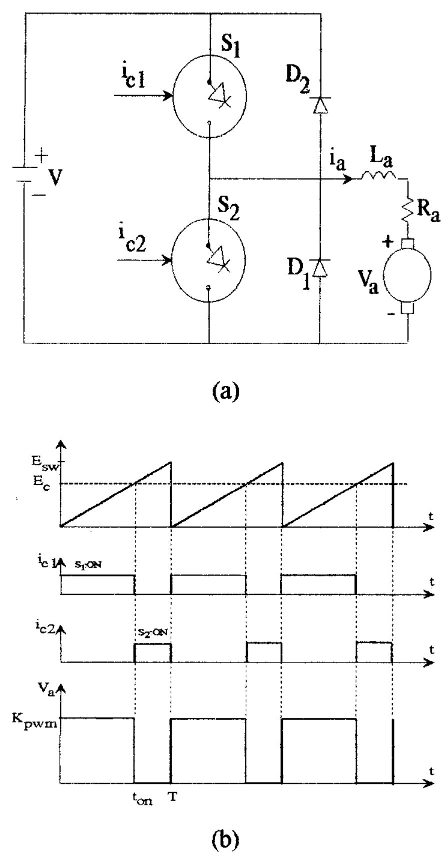

Figure 3. Class-C chopper circuit and related waveforms

$$t_{on} = T \qquad \text{for} \quad E_c > E_{sw} \tag{11}$$

Therefore, the problem in the simulation of the system is to generate the real time modeling of the PWM waveform in each period as a function of the duty cycle(δ) defined as t_{on}/T. This can be accomplished via Matlab if the z-transformation of the PWM waveform during a period is known. If it is assumed that the PWM signal is high at the begining of a period, then the z-transform of the PWM waveform over one period whose amplitude is K_{pwm} Volts can be written by using the definition of z-transform as given below [3]

$$V_a(z) = K_{pwm} \cdot \left\{ 1 \cdot z^{0} + 1 \cdot z^{-1} + 1 \cdot z^{-2} + \cdots + 1 \cdot z^{-(nos-1)} + 0 \cdot z^{-nos} + \cdots + 0 \cdot z^{-(n-1)} \right\} \tag{12}$$

where the variable nos is the nearest integer to the value of t_{on}/T_s, T_s is the sampling period of z-transformation, and n is the number of samples in one period.

Equation (12) can be rewritten as given below

$$V_a(z) = K_{pwm} \cdot \left\{ \frac{1 \cdot z^{n-1} + 1 \cdot z^{n-2} + 1 \cdot z^{n-3} + \cdots + 1 \cdot z^{n-nos} + 0 \cdot z^{n-nos-1} + \cdots + 0 \cdot z^{0}}{z^{n-1}} \right\} \tag{13}$$

Thus the PWM waveform is generated by using the inverse z-transform of Eq(13), which is the advantage over Eq(12), in a dedicated program written in Matlab[4].

2.3. Modeling of the Controllers

There are two controllers in the system one for current and one for speed control. For both of the speed and current controllers, proportional-integral (PI) control is chosen. Transfer function of a PI controller in z-domain according to the trapezoidal integration rule is as given below in Equation (14)

$$G(z) = K_P + K_I \cdot \frac{T}{2} \cdot \frac{z+1}{z-1} \tag{14}$$

where K_P is the proportional constant, K_i is the integral constant. However, in this study, a unit delay term is used to describe all the computational and loop delays that may occur, thus the current controller transfer function is taken as

$$G_{ci}(z) = \frac{K_{pi}}{z} + K_{ii} \frac{T}{2z} \cdot \frac{(z+1)}{(z-1)} \tag{15}$$

and the speed controller transfer function is considered as

$$G_{cs}(z) = \frac{K_{ps}}{z} + K_{is} \frac{T}{2z} \cdot \frac{(z+1)}{(z-1)} \tag{16}$$

where K_{pi} and K_{ps} are the proportional constants for current and speed controllers, respectively. Similarly K_{ii} and K_{is} are the integral constants for the current and speed controllers, respectively.

In this study, these controller parameters which keep the system in stable operating region

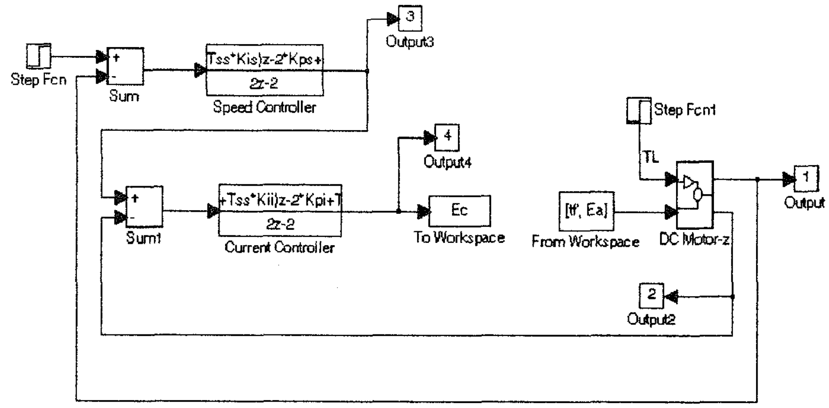

Figure 4. Block diagram representation of the Simulink model

are chosen as K_{ps}=1, K_{is}=5, K_{pi}=10 and K_{ii}=500 with the help of the root-locus method.

3. SIMULATION RESULTS

Simulation has been performed with the help of the software named Simulink[5]. The Simulink model of the system used for the simulation is given in Figure 4. The discrete transfer functions of the speed and current controllers in the Simulink model are given in Equations (15) and (16). The blocks "Step Fcn" and "Step Fcn1" represent the reference speed and the load torque respectively. The subsystem named "DC Motor-z" consists of the transfer functions given in Figure 2.

The PWM waveform at the output of the chopper has been generated by comparing the output of current controller to the sawtooth waveform. This comparison is performed by receiving the value of E_c from the simulation program. The computation of the duty cycle depending on this value and peak value of sawtooth wave is made in a dedicated program written in Matlab. After receiving the duty cycle as a result of this computation, the PWM waveform has been generated as mentioned in Section 2.2 and the data is transfered to the Simulink in order to continue to the simulation of the system. This communication between Simulink and dedicated program is carried out via the blocks (To Workspace and From Workspace) given in Figure 4 once in every sampling period.

In this study, 110V, 2.5 hp, 1800 rpm a separately excited DC motor having the following parameters is used: R_a=1 ohm, L_a=46 mH, J=0.093 kgm^2, B_v=0.008 Nt-m/rd/sec, $K\varphi$=0.55 V/rad/sec. The other parameters used in the simulation are as follows: Amplitude of PWM armature voltage (K_{pwm})=110 V, Peak value of the sawtooth waveform (E_{sw})=12 V, Reference speed (w_{ref})=80 rad/sec, Load torque (T_L)=0. In addition, the linear gains of current and speed transducers (k_1 and k_2) have been chosen unity.

Figure 5 shows the variation of the armature current and rotor speed in time obtained from the simulation of the system for the chopping frequency of 1 KHz. Figure 6 shows the change of the armature current and the rotor speed obtained from the average value model. The ripple on the current waveform in Figure 5 is a result of the discrete modeling of PWM, however, the smooth variation of the same waveform in Figure 6 reflects the continuous change of armature voltage in average value modeling. Figure 7 shows the PWM waveform at the armature terminal generated from Equation (13) in simulation.

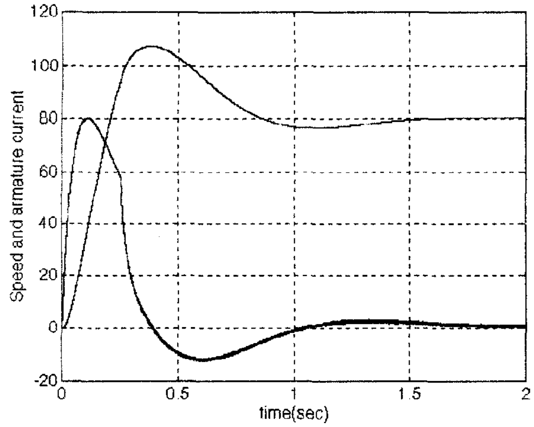

Figure 5. Results from the model in z-domain

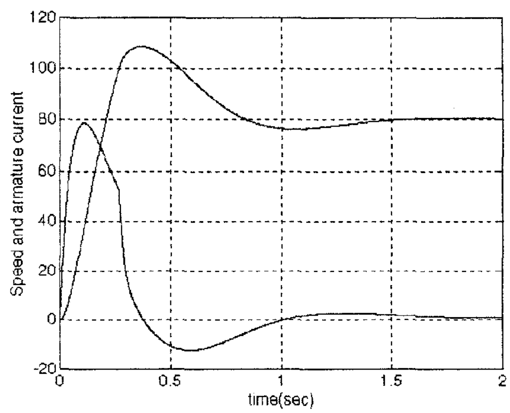

Figure 6 Results from the average value model

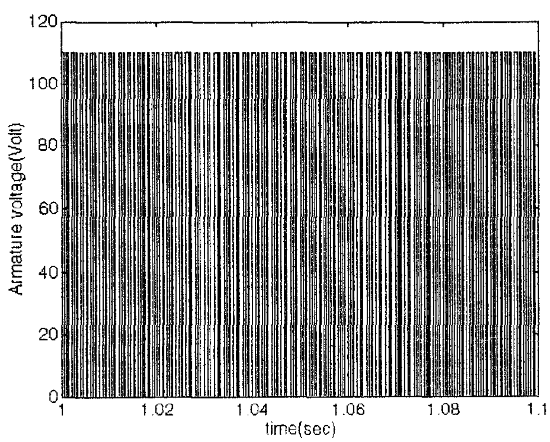

Figure 7. PWM waveform at the armature terminal

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